Introduction to Euclid's Geometry

- **Definitions in Euclidean Geometry:**

  During Euclid's period, the notations of points, line, plane (or surface), and so on were derived from what was seen around them.

  Some of the definitions given in him are as follows:

  o A **point** is that which has no part.

  o A **line** is breadth-less length.

  o **A straight line** is one that lies evenly with the points on itself.

  o A **surface** is that which has length and breadth only.

  o The **edges** of a surface are lines.

  o A **plane surface** is one that lies evenly with the straight lines on itself.

- **Euclid's axioms:**
  Axioms are the assumptions that are obvious universal truths, but are not proved. These are used throughout mathematics and are not specifically linked to geometry.

  Some of Euclid's axioms are as follows:

  o Things that are equal to the same things are equal to one another.

  o If equals are added to equals then the wholes are also equal.

  o If equals are subtracted from equals then the remainders are equal.

  o Things that coincide with one another are equal to one another.

  o The whole is greater than the part.

  o Things that are double of the same things are equal to one another.

  o Things that are halves of the same things are equal to one another.

- **Euclid's postulates:**
  Postulates are also universal truths that need not be proved. Euclid used the term "postulate" for the assumptions that were specific to geometry.

Postulate 1: It is possible to draw a straight line from any point to any other point. Euclid has frequently assumed this postulate, without mentioning that there is a **unique** line joining two distinct points. The above result can be stated in the form of an axiom as follows.

Axiom: Given two distinct points, there is a unique line that passes through them.

Postulate 2: A terminated line can be produced indefinitely.

Note: According to present day terms, the second postulate states that a line segment can be extended on either side to form a line.

Postulate 3: It is possible to describe a circle with any centre and radius.

Postulate 4: All right angles are equal to one another.

- **Postulate 5:**
  If a straight line falling on two straight lines forms interior angles that together measure less than two right angles on the same side of it, then the two straight lines, when produced indefinitely, meet on that side on which the sum of the angles is less than two right angles. Euclid's fifth postulate is also called **parallel postulate**.

  Two cases arise from this postulate:

  Case 1: If the sum of the two angles is greater than two right angles then the two lines will never meet, no matter how long they are extended in that direction.

  Case 2: If the sum of the two angles is exactly equal to two right angles then the two lines will never meet, no matter how long they are continued in both directions.

- Playfair's axiom is the alternative of Euclid's fifth postulate, which can be stated as:
  For every line in a plane, there exists another unique line in the same plane parallel to the given line, which passes through a point placed at a certain distance from that line.